Central University of Punjab, Bathinda

Course Scheme & Syllabus
for
Ph. D. Course Work
in
Mathematics / Statistics

Syllabi applicable for Admissions in Ph. D. (Mathematics/Statistics), 2016
Students can move into the Ph.D. programme after successful completion of one semester coursework, provided they meet the requirements specified by the university.

**Structure for Ph. D. course work in Mathematics / Statistics**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Subject Code</th>
<th>Subject Name</th>
<th>Credit Hours</th>
<th>Maximum Marks</th>
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<td>Lecture</td>
<td>Practical</td>
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<tr>
<td>1.</td>
<td>PHDMS.701</td>
<td>Research Methodology and Computer Applications</td>
<td>4</td>
<td>4</td>
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<tr>
<td>2.</td>
<td>PHDMS.702</td>
<td>Review Writing and Presentation</td>
<td>8</td>
<td>4</td>
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<td>3.</td>
<td>PHDMS.703</td>
<td>Real Analysis</td>
<td>4</td>
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<td>4.</td>
<td>PHDMS.704</td>
<td>Linear Algebra</td>
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<td>5.</td>
<td>PHDMS.705</td>
<td>Symmetries and Differential Equations</td>
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<td>6.</td>
<td>PHDMS.706</td>
<td>Fractional Calculus</td>
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<td>7.</td>
<td>PHDMS.707</td>
<td>Advanced Partial Differential Equations</td>
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<td>8.</td>
<td>PHDMS.708</td>
<td>Advanced Numerical Analysis</td>
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<td>9.</td>
<td>PHDMS.709</td>
<td>Operations Research</td>
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<td>10.</td>
<td>PHDMS.710</td>
<td>Number Theory</td>
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<td>11.</td>
<td>PHDMS.711</td>
<td>Advanced Algebra</td>
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<td>12.</td>
<td>PHDMS.712</td>
<td>Functional Analysis</td>
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<td>13.</td>
<td>PHDMS.713</td>
<td>Advanced Complex Analysis</td>
<td>4</td>
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<tr>
<td>14.</td>
<td>PHDMS.714</td>
<td>Differential Equations and Boundary-Value Problems</td>
<td>4</td>
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<tr>
<td>15.</td>
<td>PHDMS.715</td>
<td>Potential Flow of Fluids and Wave Theory</td>
<td>4</td>
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<td>16.</td>
<td>PHDMS.716</td>
<td>Topology</td>
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<td>17.</td>
<td>PHDMS.717</td>
<td>Differential Geometry of Curves and Surfaces</td>
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<td>PHDMS.718</td>
<td>Differential Topology</td>
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<td>PHDMS.719</td>
<td>Algebraic Topology</td>
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<td>PHDMS.720</td>
<td>Riemannian Geometry</td>
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<td>PHDMS.721</td>
<td>Finsler Geometry</td>
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<td>22.</td>
<td>PHDMS.722</td>
<td>Lie Groups and Lie Algebra</td>
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<td>23.</td>
<td>PHDMS.723</td>
<td>Probability Theory</td>
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<td>24.</td>
<td>PHDMS.724</td>
<td>Stochastic Processes and Queuing Theory</td>
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<td>25.</td>
<td>PHDMS.725</td>
<td>Reliability Theory</td>
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<td>26.</td>
<td>PHDMS.726</td>
<td>Sampling Theory</td>
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Centre for Mathematics and Statistics, CUPB

Syllabi for PhD Course work

Semester I

Course Title: Research Methodology and Computer Applications

Course Code: PHDMS.701

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Total Hours: 60

Course Objective: The objective of this course is to ensure that a student learns basis of scientific research. Also the student will develop understanding of computer hardware systems and its few basic applications.

Unit-I (15 Lecture Hours)


Literature Survey and Review: Meaning of Literature Survey and Review, Sources of Literature, Methods of Literature Review, and Techniques of Writing the Reviewed Literature.


Unit-II (15 Lecture Hours)


Unit III (15 Lecture Hours)

Fundamentals of Computers and Internet: Block Diagram of computer, Hardware components, Introduction to computer network and world wide web, Sharing data over network, Internet terminology, Searching over internet, Google: advance search operations, Email, Checking Plagiarism using internet

Unit IV (15 Lecture Hours)

Introduction to Microsoft Office: Creating and saving documents, Text formatting, Tables, Document review option, Mail merge, Inserting table of contents, Reference management. Introduction to spreadsheet and Microsoft excel, Text formatting, Formulas, Charts, Table formatting, Sorting records, Filtering the content. Introduction to Microsoft power point, Layout selection, Designing and formatting slides, Slide design and background formatting,

Recommended Books:
Course Title: Review Writing and Presentation

Course Code: PHDMS.702

Total Hours: 120

Objective: The objective of this course would be to ensure that the student learns the aspects of the Review writing and seminar presentation. Herein the student shall have to write a review of existing scientific literature with simultaneous identification of knowledge gaps that can be addressed through future work.

The evaluation criteria for “Review Writing and Presentation” shall be as follows:

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<th>S. No.</th>
<th>Criteria</th>
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<tr>
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<td>Review of literature</td>
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<td>Identification of gaps in knowledge</td>
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<td>Presentation Skills</td>
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<td>6.</td>
<td>Handling of queries</td>
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Total 100
Course Title: Real Analysis
Course Code: PHDMS.703
Total Hours: 60

Objective: The aim of this course is to make the students learn fundamental concepts of metric spaces, the Riemann-Stieltjes integral as a generalization of Riemann Integral, Sequence and series of functions, the calculus of several variables and some basic theorems.

Unit-I  
Set Theory: Finite, countable and uncountable sets
Metric spaces: Definition and examples, Open and closed sets, Compact sets, Elementary properties of compact sets, k- cells, Compactness of k-cells, Compact subsets of Euclidean space $\mathbb{R}^k$, Perfect sets, Cantor set, Separated sets, Connected sets in a metric space, Connected subsets of real line.

Unit-II  
Sequences in Metric spaces: Convergent sequences, Subsequences, Cauchy sequences, Complete metric space, Cantor’s intersection theorem, Category of a set and Baire’s category theorem. Examples of complete metric space, Banach contraction principle.

Unit-III  
Continuity: Limits of functions (in Metric spaces), Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Uniform continuity.
Riemann Stieltje’s Integral: Definition and existence of Riemann Stieltje’s integral, Properties of integral. Integration and Differentiation. Fundamental Theorem of Calculus, 1st and 2nd Mean Value Theorems of Riemann Stieltje’s integral.

Unit-IV  
Sequences and series of functions: Problem of interchange of limit processes for sequences of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, equicontinuous families of functions, Stone Weierstrass Theorem.

Recommended Books:
Course Title: Linear Algebra
Course Code: PHDMS.704

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Total Hours: 60

Objective:
The concepts and techniques from linear algebra are of fundamental importance in many scientific disciplines. The main objective is to introduce basic notions in linear algebra that are often used in mathematics and other sciences. The emphasis will be to combine the abstract concepts with examples in order to intensify the understanding of the subject.

Unit I
(14 Lecture Hours)
Vector Space: Vector spaces, Subspaces, Direct sum of subspaces, Linear dependence and independence, Basis and dimensions, Linear transformations, Algebra of linear transformations, Dual spaces, Matrix representation of a linear transformation, Rank and nullity of a linear transformation, Invariant subspaces. Change of basis.

Unit I
(15 Lecture Hours)
Characteristic polynomial and minimal polynomial of a linear transformation, Cayley Hamilton theorem, Eigenvalues and eigenvectors of a linear transformation, Diagonalization and triangularization of a matrix, Characteristic polynomial and minimal polynomial of block matrices. Canonical forms, Diagonal forms, Triangular forms, Jordan canonical forms, rational canonical forms, Quotient spaces.

Unit III
(14 Lecture Hours)
Linear functional, Dual space, Dual basis, Annihilators, Bilinear forms, Symmetric bilinear forms, Sylvester’s theorem, quadratic forms, Hermitian forms. Reduction and classification of quadratic forms.

Unit IV
(13 Lecture Hours)

Recommended books:
4. V. Bist and V. Sahai, Linear Algebra, Narosa, Delhi, 2002.
Course Title: Symmetries and Differential Equations  
Course Code: PHDMS.705  
Total Hours: 60

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<th>Course Title</th>
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Unit I  
**Dimensional Analysis**: Buckingham Pi-theorem, Assumptions behind dimensional analysis, Conclusions from dimensional analysis, Proof of the Buckingham Pi-theorem and examples, Application of dimensional analysis to partial differential equations, Generalization of dimensional analysis, Invariance of partial differential equations under scaling of variables

Unit II  
**Lie Group of Transformations**: Groups, Examples of groups, Groups of transformations, One-parameter Lie group of transformations, Examples of one-parameter Lie groups of transformations, Infinitesimal transformations: First fundamental theorem of Lie, Infinitesimal generators, Invariant functions

Unit III  
Canonical coordinates, Invariant surfaces, Invariant curves, Invariant points, Extended transformations: Extended group transformations-one dependent and one independent variable, Extended infinitesimal transformations-one dependent and one independent variable, Extended transformations-one dependent and n independent variables

Unit IV  
Multi-parameter Lie groups of transformations; Lie algebras, r-parameter Lie groups of transformations, Lie algebras, Examples of Lie algebras, Solvable Lie algebras

**Recommended Books:**

Course Title: Fractional Calculus  
Course Code: PHDMS.706  
Total Hours: 60

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Unit I (16 Lecture Hours)
Special Functions of Fractional Calculus: Gamma function, Some properties of Gamma function, Beta function, Contour integral representation. Fractional derivatives and integrals, Grunwald-Letnikov Fractional derivatives, Riemann-Liouville fractional derivatives, Caputo’s fractional derivative, The Leibniz rule for fractional derivatives, Geometric and physical interpretation of fractional integration and fractional differentiation.

Unit II (14 Lecture Hours)

Unit III (15 Lecture Hours)

Unit IV (15 Lecture Hours)

Recommended Books:
Course Title: Advanced Partial Differential Equations

Course Code: PHDMS.707

Total Hours: 60

Objectives:
The objective of this course is to equip the students with knowledge of some advanced concepts related to partial differential equations and to understand some basic approaches to mathematical oriented PDEs.

Unit-I (16 Lecture Hours)
Distribution: Test Functions and Distributions, Examples, Operations on Distributions, Supports and Singular Supports, Convolution, Fundamental Solutions, Fourier Transform, Schwartz space, Tempered Distributions.

Sobolev Spaces: Basic properties, Approximation by smooth functions, Extension theorems, Compactness theorems, Dual spaces, Functional order spaces, Trace spaces, Trace theory, Inclusion theorem.

Unit-II (15 Lecture Hours)

Unit-III (14 Lecture Hours)
Evolution Equations: Unbounded linear operators, $C_0$ – Semigroups, Hille-Yosida theorem, Contraction Semigroup on Hilbert Spaces, Heat equation, Wave equation, Schrodinger equation, Inhomogeneous equations.

Unit-IV (15 Lecture Hours)

Recommended Books:
**Course Title:** Advanced Numerical Analysis

**Course Code:** PHDMS.708

**Total Hours:** 60

**Objectives:** The objective of the course is to familiarize the students about some advanced numerical techniques e.g. solving systems of nonlinear equations, linear system of equations, Eigen value problems, Interpolation and Approximation techniques and their use in differentiation and integration, differential equations etc.

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**UNIT- I**

**Non-Linear Equations:** Methods for multiple roots, Muller’s, Iteration and Newton-Raphson method for non-linear system of equations, and Newton-Raphson method for complex roots.

**Polynomial Equations:** Descartes’ rule of signs, Birge-Vieta, Bairstow and Giraffe’s methods.

**System of Linear Equations:** Triangularization, Cholesky and Partition methods, SOR method with optimal relaxation parameters.

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**UNIT-II**

**Eigen-Values of Real Symmetric Matrix:** Similarity transformations, Gerschgorin’s bound(s) on eigenvalues, Givens, Householder and Rutishauser methods.

**Interpolation and Approximation:** B - Spline and bivariate interpolation, Gram-Schmidt orthogonalisation process and approximation by orthogonal polynomial, Legendre and Chebyshev polynomials and approximation.

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**UNIT- III**

**Differentiation and Integration:** Differentiation and integration using cubic splines, Romberg integration and multiple integrals.

**Ordinary Differential Equations:** Shooting and finite difference methods for second order boundary value problems, Applications of cubic spline to ordinary differential equation of boundary value type.

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**UNIT- IV**

**Partial Differential Equations:** Finite difference methods for Elliptic, Parabolic and Hyperbolic partial differential equations.

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**Recommended Books:**

Course Title: Operations Research
Course Code: PHDMS.709

Total Hours: 60

Objective:
The objective of this course is to acquaint the students with the concept of convex sets, their properties, Linear and nonlinear programming problems. The results, methods and techniques contained in this paper are very well suited to the realistic problems in almost every area

Unit-I (15 Lecture Hours)

Unit-II (15 Lecture Hours)

Unit-III (15 Lecture Hours)

Unit -IV (15 Lecture Hours)
Replacement problem, replacement of items that Deteriorate, replacement of items that fails completely. Job Sequencing Problems; Introduction and assumption, Processing of n jobs through two machines, Processing of n jobs through three machines and m machines, Processing two jobs through n machines.

Recommended books:
Course Title: Number Theory
Course Code: PHDMS.710
Total Hours: 60

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Objective:
The objective of this course is to teach the fundamentals of different branches of Number Theory, namely, Geometry of Numbers and Analytic Number Theory.

Unit I  
Divisibility of Integers, Greatest common divisor, Euclidean algorithm. The fundamental theorem of arithmetic, Congruences, Residue classes and reduced residue classes.

Unit II  
Indices and its applications, Quadratic residues, Euler’s criterion, Product of quadratic residues and quadratic non-residues, The Legendre symbol and its properties, Gauss’s lemma, Quadratic reciprocity law, Jacobi symbol and its properties.

Unit III  
Chinese remainder theorem, Fermat’s little theorem, Wilson’s theorem, Euler’s theorem. Arithmetic functions σ(n), d(n), τ(n), μ(n), Order of an integer modulo n, primitive roots for primes, composite numbers having primitive roots.

Unit IV  
Representation of an integer as a sum of two and four squares. Diophantine equations ax +by =c, x2+y2=z2 and its application to x4+y4=z4. Farey sequences, Continued fractions.

Recommended books:
Course Title: Advanced Algebra
Course Code: PHDMS.711
Total Hours: 60

Objective:
This course is for students who wish to pursue research work in Algebra.

Unit I
Groups: Jordan Holder theorem; solvable groups; symmetric and alternating groups; nilpotent groups; groups acting on sets; Sylow theorems; free groups.

Unit-II
Rings and Modules: Noetherian and Artinian rings and modules; semi-simple rings; Hilbert basis theorem; Principal ideal domains and unique factorisation domains; modules over PID; linear algebra and Jordan canonical form; structure theorems for semi-simple rings. Representation theory of finite groups.

Unit-III
Field Theory: Basic concepts of field theory, Steinitz theorem, Extension of fields, algebraic and transcendental extensions. Algebraically closed fields, Splitting fields, Separable and inseparable extensions, Normal extension, Multiple roots, Finite fields, Perfect fields.

Unit-IV
Galois Theory: Automorphism groups, Fixed fields, Galois extensions, The fundamental theorem of Galois theory, Cyclotomic extensions, and Cyclic extensions, Applications of cyclotomic extensions and Galois theory to the constructability of regular polygons, Solvability of polynomials by radicals.

Recommended Books:
4. V. Bist and V. Sahai, Linear Algebra, Narosa, Delhi, 2002.
Course Title: Functional Analysis
Course Code: PHDMS.712
Total Hours: 60

Objective: The objective of this course is to introduce basic concepts, methods of Functional Analysis and its Applications. It is a first level course in Functional Analysis.

Unit-I
Fundamentals of Normed Linear Spaces: Normed Linear spaces, Banach spaces and examples, finite dimensional normed spaces and subspaces, compactness and finite dimension. Quotient space of normed linear spaces and its completeness.

Unit-II
Weak convergence and bounded linear transformations, Normed linear spaces of bounded linear transformations, Dual spaces with examples.

Three Main Theorems on Banach Space: Uniform boundedness theorem and some of its consequences, Open mapping and closed graph theorems.

Unit-III

Unit-IV

Recommended books:
Course Title: Advanced Complex Analysis
Course Code: PHDMS.713
Total Hours: 60

Objectives:
This course is designed to enable the readers to understand further deeper topics of Complex Analysis and will provide basic topics needed for students to pursue research in pure Mathematics.

Unit–I
Harmonic function: Definition, Relation between a harmonic function and an analytic function, Examples, Harmonic Conjugate of a harmonic function, Poisson's Integral formula, Mean Value Property, The maximum & minimum principles for harmonic functions, Dirichlet Problem for a disc and uniqueness of its solution, Characterization of harmonic functions by mean value property.

Unit–II

Unit–III
Weierstrass Elliptic functions: Periodic functions, Simply periodic functions, fundamental period, Jacobi's first and second question, Doubly periodic functions, Elliptic functions, Pair of Primitive Periods, Congruent points, First and Second Liouville's Theorem, Relation between zeros and poles of an elliptic function, Definition of Weierstrass elliptic function \( z \) and their properties, The differential equation satisfied by \( (z) \), Integral formula for \( (z) \), Addition theorem and Duplication formula for \( (z) \).

Unit–IV
Weierstrass Zeta function: Weierstrass Zeta function and their properties, Quasi periodicity of \( (z) \), Weierstrass sigma function \( (z) \) and their properties, Quasiperiodicity of \( (z) \), Associated sigma functions.

Recommended Books:
Course Title: Differential Equations and Boundary-Value Problems

Course Code: PHDMS.714

Total Hours: 60

<table>
<thead>
<tr>
<th>Unit I</th>
<th>(16 Lecture Hours)</th>
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<tr>
<td>Existence and uniqueness of solutions of ODEs, Power series solution, Singular points, Some special functions. Nonlinear system of ODE : Preliminary concepts and definitions, The fundamental existence-uniqueness results, Dependence on initial conditions and parameters, The maximum interval of existence.</td>
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<th>Unit II</th>
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<th>Unit III</th>
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<td>First-order PDEs, Cauchy problem, Method of characteristics, Second-order PDEs, Classification, Characteristics and canonical forms. Elliptic boundary value problems : Maximum principle, Green's function,</td>
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<tr>
<th>Unit IV</th>
<th>(15 Lecture Hours)</th>
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Recommended Books:

**Course Title:** Potential Flow of Fluids and Water-Wave Theory  
**Course Code:** PHDMS.715  
**Total Hours:** 60

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**Unit I**  
(16 Lecture Hours)  
Real fluids and ideal fluids, Velocity of fluid at a point, Streamlines, Path lines, Streak lines, Velocity potential, Vorticity vector, Local and particle rate of change, Equation of continuity, Irrigational and rotational motion, Acceleration of fluid, Conditions at rigid boundary.

**Unit II**  
(14 Lecture Hours)  
Euler’s equation of motion, Bernoulli’s equation and their applications, Potential theorems, Axially symmetric flows, Impulsive motion, Kelvin’s Theorem of circulation, Equation of vorticity.

**Unit III**  
(16 Lecture Hours)  

**Unit IV**  
(14 Lecture Hours)  
Theory of surface wave, Finite amplitude wave, One dimensional tidal dynamics, Linear and non-linear diffraction theory, Permutation methods. Water wave interaction with submerged spherical structures and floating cylinders, Solitary waves, Cnoidal wave, Schrodinger equation.

**Recommended Books:**
**Objective:** The course is an introductory course on point-set topology. It is designed in such a way that the students can have deep knowledge in general topology and can understand further deeper topics in Differential/Algebraic Topologies and their allied areas.

**Unit I** (16 Lecture Hours)

**Unit II** (17 Lecture Hours)
Connected spaces, Connected subspaces of the real line, Components and path components, Local connectedness. Compact spaces, Sequentially compact spaces, Heine-Borel Theorem, Compact subspaces of the real line, Limit point compactness, Local compactness and one point compactification.

**Unit III** (15 Lecture Hours)

**Unit IV** (12 Lecture Hours)
The Stone–Čech Compactification, Complete metric spaces, Compactness in metric spaces, Pointwise and compact convergence, Ascoli’s Theorem and Baire spaces.

**Recommended Books:**
Course Title: Differential Geometry of Curves and Surfaces
Course Code: PHDMS.717

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Total Hours: 60

Objective: To introduce students to the local and global theory of curves and surfaces so that they can be enabled for further studies and research in Differentiable Manifolds, Differential Topology, Algebraic Topology, Riemannian Geometry.

Unit-I (15 Lecture Hours)
Curves in Plane and Space: Parameterized curves, Tangent vector, Arc length, Reparametrization, Regular curves, Curvature and Torsion of smooth curves, Frenet-Serret formulae, arbitrary speed curves, Frenet approximation of a space curve. Osculating plane, Osculating circle, Osculating sphere, Involutes and Evolutes, Bertrand curves, Spherical indicatrices, Helices, Fundamental theorem of space curves,

Unit-II (15 Lecture Hours)
Isomeries of $R^3$, Congruence of curves. Surfaces in $R^3$: Definition and examples, Smooth surfaces, Tangent, Normal and Orientability. Examples of surfaces: Generalized cylinder and generalized cone, Ruled surfaces. Surface of revolution and Quadric surfaces. First fundamental form, Isometry of surfaces, Conformal mapping of surfaces, Surface Area, Equi-areal maps and a Theorem of Archemedes,

Unit-III (15 Lecture Hours)
Second fundamental form, Curvature of curves on a surface, Normal and Principal curvatures, Meusnier’s theorem, Euler’s theorem, Weingarten equations and Weingarten map. Geometric interpretation of principal curvatures, Umbilical points. Gaussian and Mean curvature, Pseudo Sphere, Flat surfaces. Surfaces of constant mean curvature, Gaussian curvature of compact surfaces, Gauss map.

Unit-IV (15 Lecture Hours)
Geodesics: Definition and basic properties, Geodesic equations, Geodesics on a surfaces of revolution, Clairaut’s theorem, Geodesics as shortest paths, Geodesic coordinates, Gauss Theorem, Egregium, Gauss equations, Codazzi-Mainardi equations, Compact surfaces of constant Gaussian curvature.

Recommended Books:

Syllabi applicable for Admissions in Ph. D. (Mathematics/Statistics), 2016
Course Title: Differential Topology
Course Code: PHDMS.718
Total Hours: 60

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Objective: To introduce students to Differential Topology so that they can be enable for pursuing research in Differential Topology, Riemannian geometry and its allied areas.

UNIT-I (15 Lecture Hours)
The standard differential structure on the Euclidean space $\mathbb{R}^n$. Definition of manifold as a submanifold of $\mathbb{R}^n$, The standard abstract definition of manifolds using transition functions; Examples including the spheres, Real projective spaces, Higher genus surfaces. Definition of orientability of a manifold with examples (To discuss why Moebius band, Real projective plane and Klein bottle are not orientable).

UNIT-II (15 Lecture Hours)
Smooth maps and diffeomorphisms, the inverse function theorem, immersion and submersion, embedding, local immersion and local submersion theorems, critical and regular points (values) of a smooth map. Support of a function, bump functions, smooth version of Urysohn's Lemma for a manifold, Partition of unity.

UNIT-III (15 Lecture Hours)

UNIT-IV (15 Lecture Hours)

Recommended Books:
Course Title: Algebraic Topology
Course Code: PHDMS.719
Total Hours: 60

Objective: The objective of this course is to introduce the student’s concept in Algebraic topology so that they can pursue research in this field and its allied areas.

Unit-I (14 Lecture Hours)

Unit-II (14 Lecture Hours)

Unit-III (16 Lecture Hours)

Unit-IV (16 Lecture Hours)

Recommended Books:
Course Title: Riemannian Geometry
Course Code: PHDMS.720
Total Hours: 60

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Objective: The objective of the course is to introduce the basic concepts of Differentiable manifolds and Riemannian geometry and also for further studies and research in Riemannian geometry, Finsler geometry, Mathematical Physics and their applications in allied areas.

Unit I (16 Lecture Hours)

Unit II (14 Lecture Hours)
Tensors and tensor fields (Riemannian metric as the most significant example), Tensorial property, Covariant differentiation of tensor fields, Riemann curvature tensor, Ricci tensor, Definition of sectional, Ricci and scalar curvatures, Isometries, notion of covering spaces, pull-back metrics via diffeomorphisms.

Unit III (16 Lecture Hours)
Covariant differentiation of a vector field along a curve with specific examples, Arc length and energy of a piecewise smooth curve, Geodesics as length minimizing curves, First variation of arc length. To show that geodesics are critical points of the fixed end point first variation formula, Exponential map, Geodesic completeness, Geodesic normal coordinates, Hopf-Renow Theorem (statement only), Geodesic variations, Jacobi fields and Gauss lemma.

Unit IV (14 Lecture Hours)
Second variation formula, the index form (Jacobi fields as minimizers of the Index form), Global differential geometry, spaces of constant sectional curvature, Bonnet-Myers, Cartan-Hadmard Theorem, Cartan’s Theorems (on determination of metric by curvature).

Recommended Books:

Syllabi applicable for Admissions in Ph. D. (Mathematics/Statistics), 2016
Course Title: Finsler Geometry
Course Code: PHDMS.721
Total Hours: 60

Objective: The objective of this course is to enable students Riemann-Finsler geometry so that they can pursue research in this area.

UNIT I (15 Lecture Hours)
Basic Concepts of a Finsler space: Line elements, Finsler space, Minkowskian space, Tangent space, Indicatrix, Metric Tensor, Dual tangent space, Hamiltonian function, Angle between two vectors, Generalized Christoffel symbols, Geodesics.

UNIT II (15 Lecture Hours)
Covariant Differentiation: $\delta$-derivative, Partial $\delta$-derivative, Fundamental postulates of E. Cartan, Different deductions, Cartan’s two processes of covariant differentiation, Berwald connection parameters, Berwald’s covariant differentiation.

UNIT III (15 Lecture Hours)
Theory of Curvature: Commutation formulae resulting from Cartan’s covariant differentiation, Cartan curvature tensor, Commutation formulae resulting from Berwald’s covariant differentiation, Berwald curvature tensor, Generalizations of Bianchi identities, Space of scalar curvature, Space of constant curvature, Generalization of Schur’s theorem, Recurrent spaces, Symmetric spaces.

UNIT IV (15 Lecture Hours)
Projective Change: Projective change, Projective invariants, Projective change of Berwald’s connection parameters, Projective deviation tensor, Generalized Weyl’s projective curvature tensor, Projective connection parameters, Projectively flat spaces, Szabó Theorem.

Recommended Books:

Course Title: Lie Groups and Lie Algebra
Course Code: PHDMS.722
Total Hours: 60

Objective:
The aim of this course is to make the students learn basic concepts of Lie groups and Lie algebra, so as to enable the students to understand further topics related to solution of differential equations.

Unit I (15 Lecture Hours)
Differential Manifolds, Topological manifolds, Charts, Atlases and smooth structure, Smooth maps and diffeomorphism, Partitions of Unity, Tangent space, Tangent map, Vector fields and 1-forms.

Unit II (15 Lecture Hours)
Lie Groups Definition and examples, Linear Lie groups, Lie group homomorphism, Lie algebra and the exponential map, Adjoint representation, Homogeneous spaces, Baker-Campbell-Housdorff formula.

Unit III (15 Lecture Hours)
Lie Algebras Definition and examples, Classical Lie algebras, Solvable and nilpotent Lie algebras, Lie and Engel theorems, Semi-simple and reductive algebras, Semi-simplicity of Classical Lie algebras

Unit IV (15 Lecture Hours)
Semisimple Lie algebras; Killing form; Jordan decomposition; Engel's Theorem, Cartan subalgebra and Root space decomposition, Geometry of Root systems, Simple roots and Weyl group, Classification of root systems; Examples

Recommended Books:
Course Title: Probability Theory
Course Code: PHDMS.723
Total Hours: 60

Objectives:
The course is designed to equip the students with various probability distributions and to develop greater skills and understanding of Sampling and Estimation.

Unit I (13 Lecture Hours)
Probability: Definition of probability-classical, Relative frequency, Statistical and axiomatic approach, Addition theorem, Boole’s inequality, Conditional probability and multiplication theorem, Independent events, Mutual and pairwise independence of events, Bayes’ theorem and its applications.

Unit II (14 Lecture Hours)

Unit III (14 Lecture Hours)
Discrete distributions: Uniform, Bernoulli, Binomial, Poisson and geometric distributions with their properties.
Continuous distributions: Uniform, Exponential, Gamma and normal distributions with their properties. Central Limit Theorem (Statement only).
Statistical estimation: Parameter and statistic, Sampling distribution and standard error of estimate. Point and interval estimation, Unbiasedness, Efficiency.

Unit IV (15 Lecture Hours)
Sampling Theory: Types of sampling, Errors in sampling, Parameter and statistic, Tests of Significance: Null Hypothesis, Alternative Hypothesis, One-tailed, Two-tailed tests. Sampling Attributes: Tests of Significance for single proportion and difference of proportions. Sampling of Variables.

Recommended books:
**Course Title:** Stochastic Processes and Queuing Theory  
**Course Code:** PHDMS.724  
**Total Hours:** 60

**Objective:** The course on Stochastic Processes and Queuing Theory, is framed to equip the students with knowledge of terms involved in Stochastic Processes as well as concepts and measures in Queuing Theory.

**Unit I**  
(15 Lecture Hours)  
Review of probability, Random variables and distributions, Generating functions and transforms; Stochastic processes, Discrete and continuous-time Markov chains, Renewal processes.

**Unit II**  
(15 Lecture Hours)  
Brownian motion; Characteristics of queueing systems, Little's formula, Markovian and non-Markovian queueing systems, Embedded Markov chain applications to M/G/1, G/M/1, and related queueing systems.

**Unit III**  
(15 Lecture Hours)  
Queues with vacations, Priority queues, Queues with modulated arrival process, Discrete-time queues and matrix-geometric methods in queues; Networks of queues, Open and closed queueing networks.

**Unit IV**  
(15 Lecture Hours)  
Algorithms to compute the performance metrics; Simulation of queues and queueing networks; Application to manufacturing, Computer and communication systems and networks.

**Recommended Books:**

Course Title: Reliability Theory
Course Code: PHDMS.725
Total Hours: 60

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Objectives:
The course on Reliability Theory is framed to equip the students with knowledge of terms involved in reliability theory as well as concepts and measures.

Unit I
Reliability Concepts and Measures: Components and systems, Coherent systems, Reliability of coherent systems, Cuts and paths, Modular decomposition, Bounds on system reliability, Structural and reliability importance of components.

Unit II
Life distributions and associated survival, Conditional survival and hazard rate functions. Exponential, Weibull, Gamma life distributions and estimation of their parameters.

Unit III
Notions of ageing. IFR IFRA, NBU, DMRL, NBUE, and HNBUE classes; their duals and relationships between them. Closures of these classes under formation of coherent systems, convolutions and mixtures.

Unit IV
Partial orderings: Convex, star, stochastic, failure rate and mean-residual life orderings. Univariate shock models and life distributions arising out of them. Maintenance and replacement policies, Availability of repairable systems.

Recommended Books:

Course Title: Sampling Theory

Course Code: PHDMS.726

Total Hours: 60

Objectives:
The course is designed to equip the students with basic knowledge of different sampling schemes, their mean and variance estimations and also give understanding of non-sampling errors.

Unit I (15 Lecture Hours)
Introduction to usual notations used in sampling. Basic finite population sampling techniques: SRSWOR, SRSWR, stratified, systematic and related results on estimation of population mean/total. Relative precision of different sampling techniques. Allocation problem in stratified sampling.

Unit II (15 Lecture Hours)
Ratio and regression estimators based on SRSWOR method of sampling. Two-stage sampling with equal size of first stage units. Double sampling for ratio and regression methods of estimation. Cluster sampling - equal clusters.

Unit III (15 Lecture Hours)
PPS WR/WOR methods [cumulative total, Lahiri’s schemes] and related estimators of a finite population mean [Thompson-Horwitz, Yates and Grundy estimator, Desraj estimators for a general sample size and Murthy’s estimator for a sample of size 2].

Unit IV (15 Lecture Hours)
Sampling and Non-sampling error with special reference to non-response problems. National sample surveys organization (NSSO) and role of various statistical organizations in national development.

Recommended Books: